Robust walking with a simple IP model

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Summary. How do robustness and energy economy in walking trade-off with each other? We address this question using concepts from Viability theory and a simple point-mass model of walking. For this model, we find all states and next-step controls such that a given desired speed can be reached without falling. We use the results to find a walking controller for Cornell Ranger that is, in some way, maximally robust and also provides energy economy. Also, our results suggest that taking larger steps is generally advantageous to cancel perturbations.

Introduction. A good walking robot has to be robust, *i.e.* able to avoid falling in most practical situations, and also use little energy to walk. However, no robot to-date is both robust (as Boston Dynamics' robots) and energy-effective (as Cornell Ranger [1]). So what are the trade-offs between these, and possibly other, desired characteristics of walking? Our goal here is to help understanding of such trade-offs and use this to help the design of robust walking controllers.

One approach is to assess all feasible robotic states with respect to different objectives. This would show areas in the state space that are more beneficial to be at with respect to, say, robustness or energetics. This approach is in the spirit of Viability theory [2], where one of the key concepts is the *viability kernel*, the set of all states of the system from which a failure can be avoided.

Here we use viability theory concepts with a simple model of walking, the Inverted Pendulum (IP) in 2D [3]. We use the results to design a robust controller for a simple model of Ranger.

Background concepts. We study walking using discrete step-to-step dynamics. We define a step to start at (the Poincaré section to be at) mid-stance, where the stance leg is vertical.

Our primary tool of analysis is *controllable* and *extended controllable regions* [4]. Controllable regions are areas in the state space. For a given motion goal, such as walking at a given speed, they show from which states the robot can, using feasible controls, reach the goal in one, two, or more steps and without falling. The n-step controllable region C_n is the set of all states from where this can be done in n steps or fewer. The limiting region C_∞ is all states from which the target can be reached eventually; C_∞ is usually almost equal to the viability kernel of the system [5].

Extended controllable regions show specific controls for the next step that allow the robot to reach the target. The extended n-step controllable region $\bar{C}_n$ is all combinations (q, u) of states q and next-step controls u, such that the robot can reach the target within n steps in total. The limiting region $\bar{C}_\infty$ is all states and next-step controls so that the target can be reached eventually.

Planar IP model. We model Ranger with the IP model in 2D. The model has two rigid massless legs and a point-mass at the hip. The swing leg can be instantaneously placed to any desired position, thus determining the step length. Collisions are assumed instantaneous and there is no double stance. Just before the collision an impulsive push-off is applied along the trailing leg.

The model has one state variable at mid-stance, velocity v, and two controls per step, the step size x_{st} and push-off magnitude p. We only consider motions forward, only walking (no flight), and that the robot reaches mid-stance at each step. Violation of these requirements is regarded as a failure. As a proxy for actuator limitations in Ranger, we also impose an upper bound on the push-off, $p < p_{max}$, and a lower bound on step-time, time from mid-stance to heel-strike (a proxy for limited hip torque in Ranger), $t_{st} > t_{st,min} > 0$. We use rough estimates for p_{max} and $t_{st,min}$ from simulations of a full model of Ranger [1]. We also bound the largest physically feasible step, $x_{st} < x_{max}$.

The extended controllable regions of the IP model are three-dimensional (one state and two control variables). For ease of presentation, we only show in Fig. 1b projections $\bar{C}_n^{x_{st}}$ of these regions onto the coordinate plane (v, x_{st}); $\bar{C}_n^{x_{st}}$ is all velocities and next step-size controls such that, with proper push-offs, the target is reachable within n steps. The target speed v_t corresponds to Ranger's 65 km walk and is approximately energy-optimal speed of walking for Ranger. For velocities and with step-sizes outside $\bar{C}_n^{x_{st}}$ the robot fails. Projections of $\bar{C}_n^{x_{st}}$ onto the velocity axis are the controllable regions C_n in Fig. 1a.

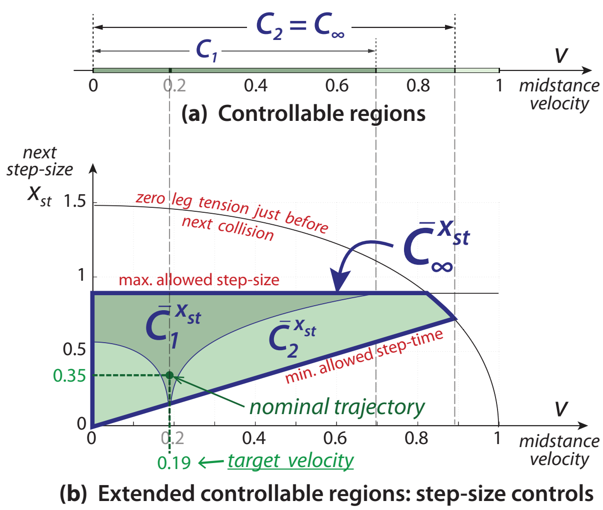

Figure 1: n-step controllability of the IP model.

Results: robust walking controller. We illustrate the design of the step-size controller here; the push-off controller design is similar, see [5].

A viable step-size controller can be defined by a function of mid-stance speed v whose graph is entirely inside the region $\bar{C}_\infty^{x_{st}}$, *e.g.* as in Fig. 2. We want a controller (a curve inside $\bar{C}_\infty^{x_{st}}$) that provides maximum robustness and also convergence to the desired target trajectory.

Robustness. We model various disturbances and errors as random changes to the robot speed v and step size x_{st}. If such random changes move the point (v, x_{st}) outside the region $\bar{C}_\infty^{x_{st}}$, the robot fails. Hence, we model robustness as distance to the boundary of $\bar{C}_\infty^{x_{st}}$: for points (v, x_{st}) that are closer to the boundary, it is more likely for the robot to fail after a disturbance. Thus, robust controllers are those farther inside the region $\bar{C}_\infty^{x_{st}}$.

Convergence. We look at how much closer the robot gets to the target after each step. We look at the ratio ν_x of the error in speed after one step to the initial error. Fig. 2 shows a color map of ν_x for all viable v and x_{st} and assuming the best viable push-off: lighter areas signify faster convergence and darker areas slower convergence or divergence from the target.

Thus, the desired step-size controller curve is inside $\bar{C}_\infty^{x_{st}}$ (viability), farther from the boundaries of $\bar{C}_\infty^{x_{st}}$ (robustness), and in lighter areas of $\bar{C}_\infty^{x_{st}}$ (convergence); it also has to pass through the target (energy-optimal nominal trajectory). We 'heuristically' pick the controller shown in Fig. 2: it is an absolute-value function whose right branch is approximately parallel to the boundary

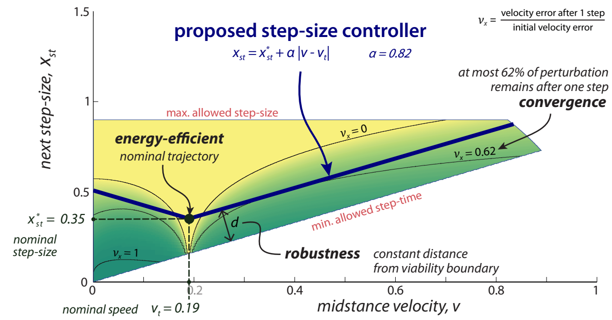

Figure 2: Robust step-size controller for Ranger.

of $\bar{C}_\infty^{x_{st}}$. With this controller, and a similarly-designed push-off controller, the robot tolerates errors of up to 48% in speed and 24% in step-size, and has an acceptable rate of convergence.

Note that our controller always prefers steps larger than the nominal, suggesting an advantage of taking larger steps for disturbance rejection.

Work in progress. The controller in Fig. 2 is manually picked and does not explicitly account for energy economy. Currently we are working on defining, and solving, a more formal optimization problem that accounts for both robustness and energetic cost. We will also consider different control horizons, *i.e.* number of steps to reach the target, thus accounting for the convergence speed.

We hope that the resulting optimal controller will help to better understand the trade-offs in bipedal walking, and also will further justify our 'Two steps is enough' result [4].

References

[1] P.A. Bhounsule. *A controller design framework for bipedal robots.* PhD thesis, Cornell University, 2012.

[2] Jean Pierre Aubin, Alexandre M Bayen, and Patrick Saint-Pierre. *Viability theory: new directions.* Springer, 2011.

[3] A.D. Kuo. Energetics of actively powered locomotion using the simplest walking model. *Journal of biomechanical engineering*, 2002.

[4] Petr Zaytsev, S Javad Hasaneini, and Andy Ruina. Two steps is enough: no need to plan far ahead for walking balance. *ICRA*, 2015.

[5] Petr Zaytsev. *Using controllability of simple models to generate maximally robust walking-robot controllers.* PhD thesis, Cornell University, 2015.